

# RISE

RESEARCH ON IMPROVING  
SYSTEMS OF EDUCATION

## Simulating learning: A formal model for learning profiles, with applications for understanding teacher value added

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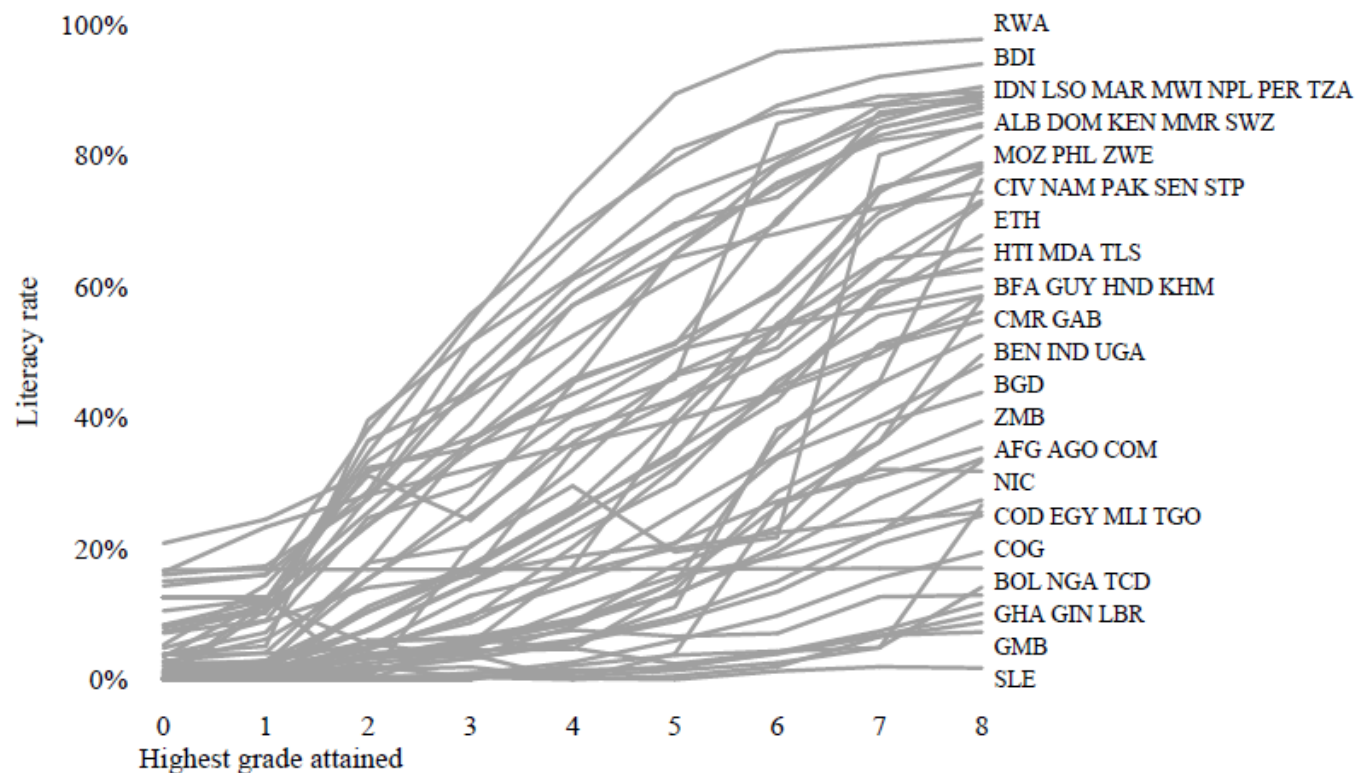
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**Washington, DC**



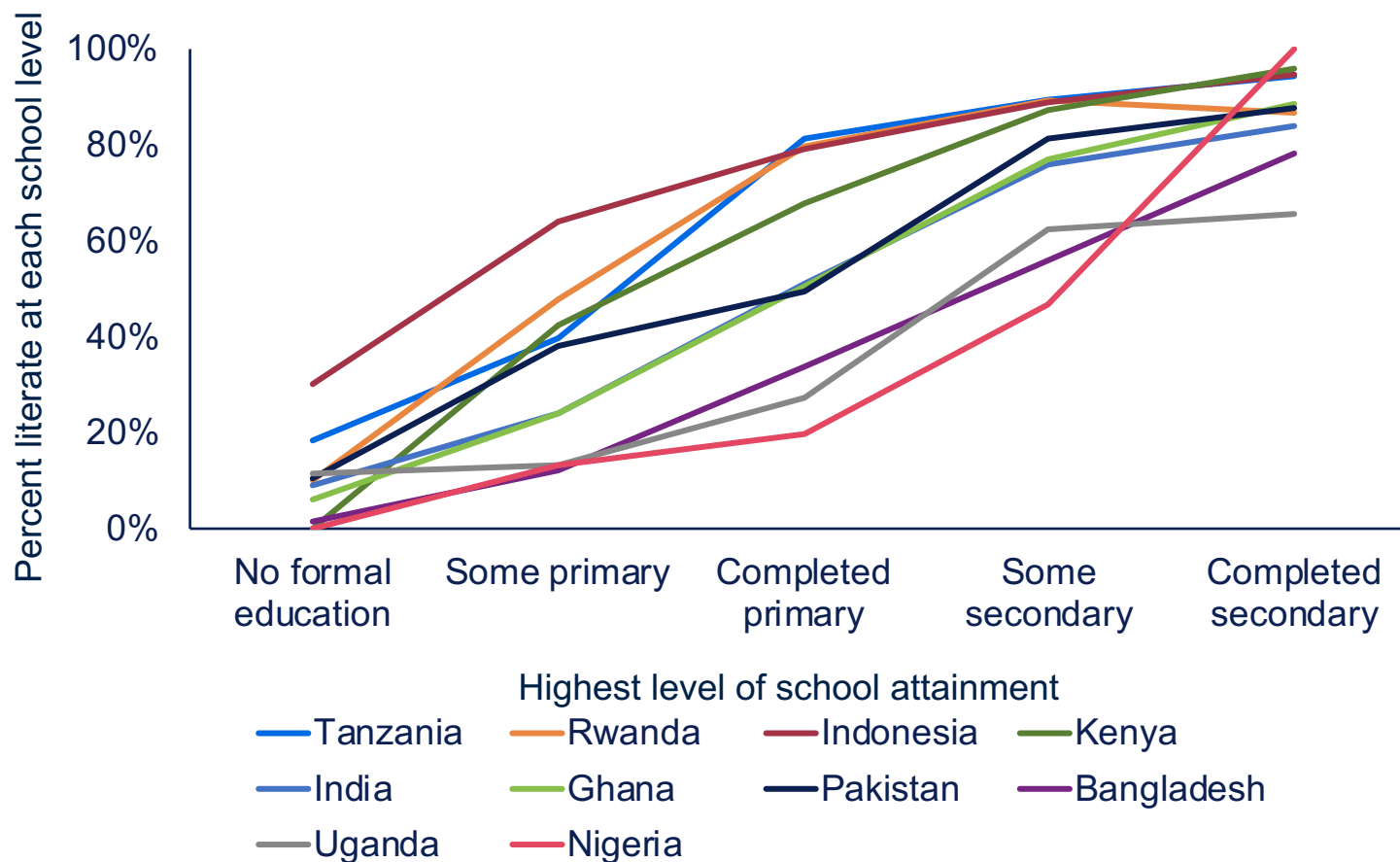
# Learning profiles show that learning is highly varied across countries and on average is low

*Learning profiles from DHS data show literacy among young women with six years of schooling varies from less than 10 percent to nearly 100 percent*



# Findings are consistent: Similar findings from different data using different literacy assessment

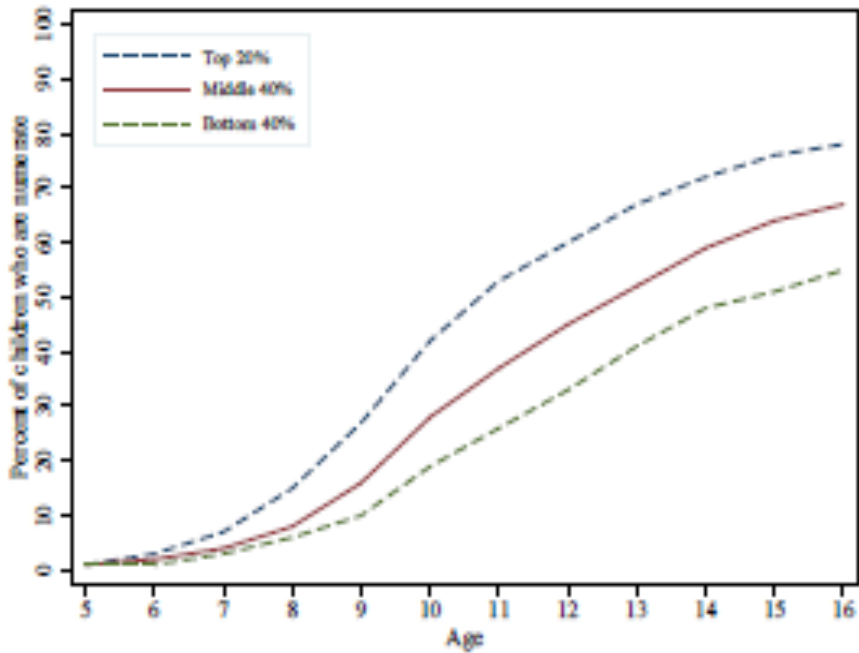
*Literacy among adults with primary completion as their highest attainment varies from 20 percent to 80 percent*



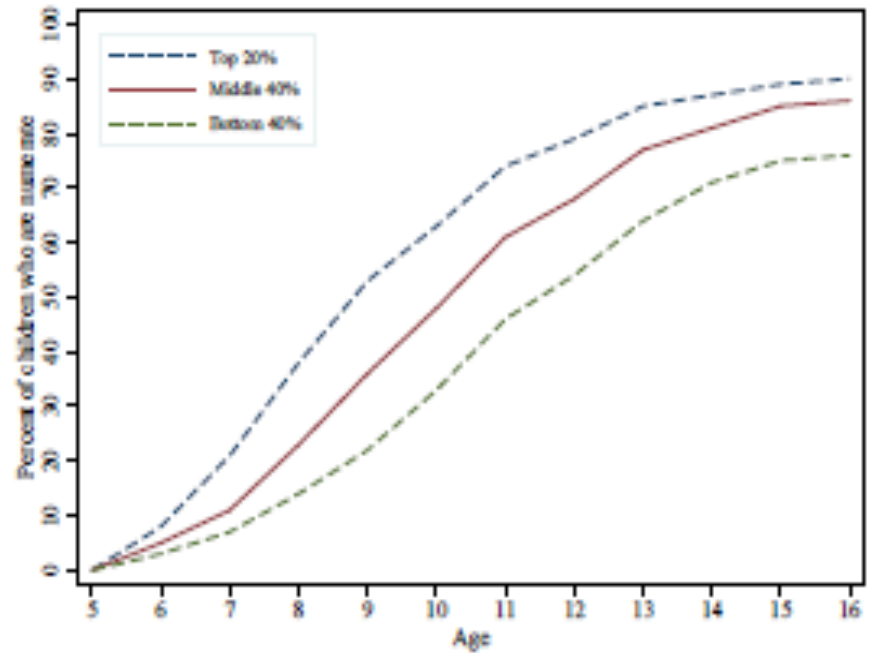
# Learning also varies substantially within countries

## Learning trajectories by age and household wealth: Math

(a) India

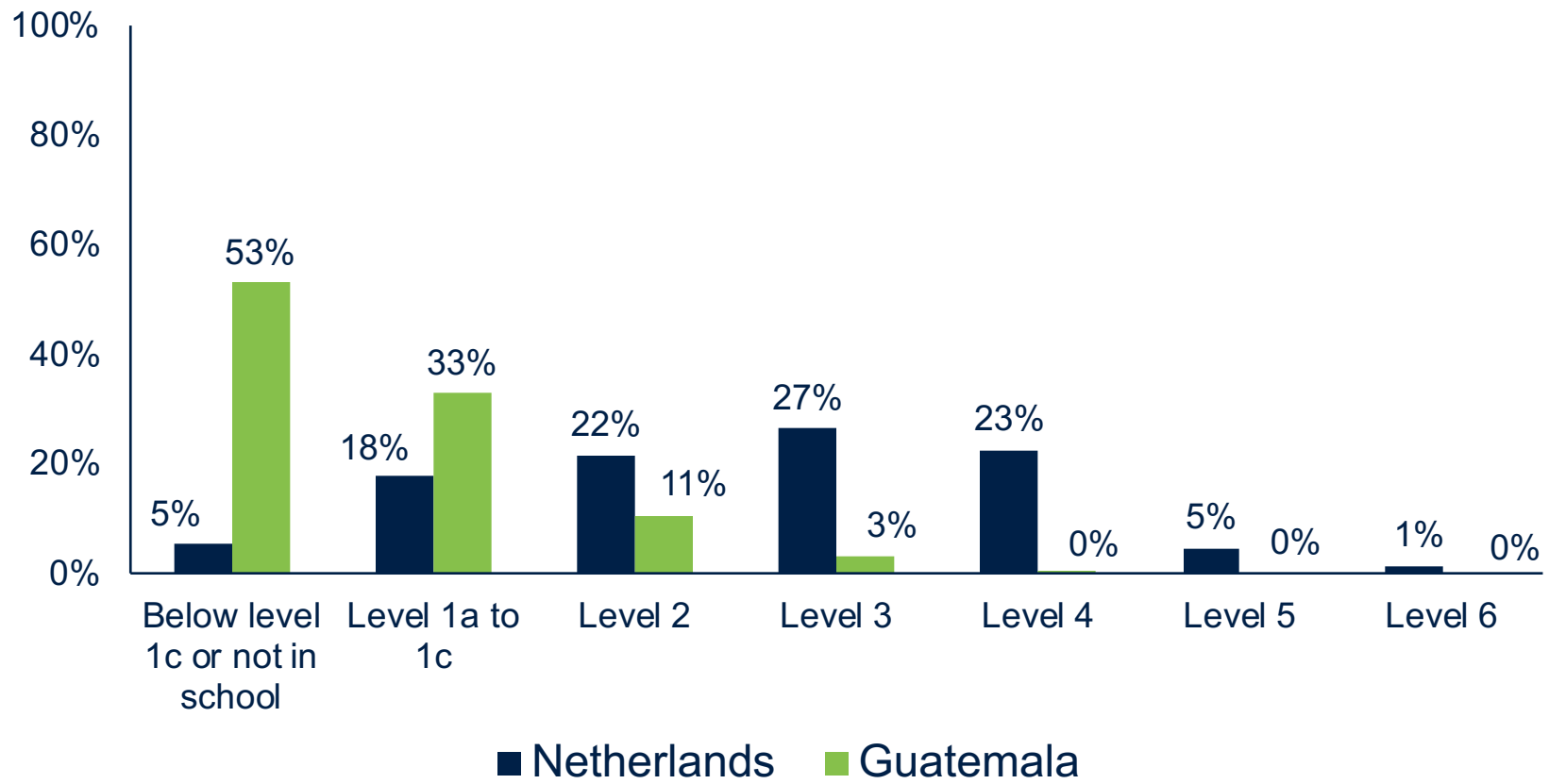


(b) Kenya



# Recent PISA-D data show developing countries are far behind any “universal basic mastery” goal

*97 percent of 15-year-olds in Guatemala are “low skill” by OECD standards, and essentially none are in the top three levels*



# How to explain learning trajectories?

Learning outcomes are driven by:

1. The initial distribution of student skills, and
2. An instructional process that imparts some level of learning for a child at each point in that distribution

# How to explain learning trajectories?

In this paper we:

- Develop a formal model that characterizes this instructional process.
- Use the model to simulate learning profiles; replicate observed learning profiles
- Show implications of the model for understanding teacher value added, curriculum pacing, and more
  - Showing that multiple factors, not just teacher ability, determine observed TVA, with implications for improving learning.

# The Potential Pedagogical Function

Building on Beatty and Pritchett (2012) we construct a potential pedagogical function (PPF)

- Models the amount a child at each point in the student distribution learns during an increment of schooling
- Characterized by five main elements:
  - **Height:** Maximum that can be learned during an increment of schooling
  - **Shape:** Determines how much children at different points in the distribution learn
  - **Range:** Range of student abilities that learn under the PPF
  - **Location/centeredness:** Level of student skills the PPF is targeted for or centered on
  - **Pace:** The amount the PPF shifts up each year (e.g. “curricular pace”)



# Potential Pedagogical Function

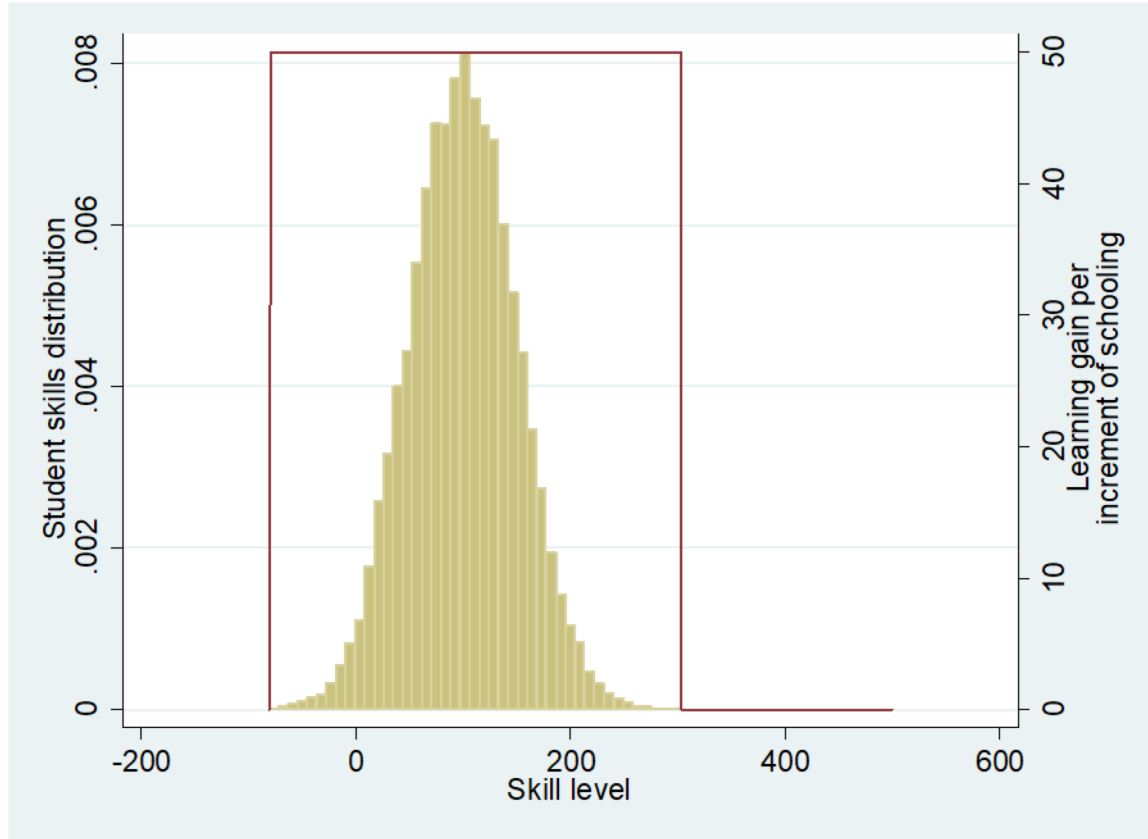
Simplest PPF, Rectangular shape, all children within the PPF range learn the same:

The learning of any student “i” of initial skill “s” is expressed as a piece-wise linear equation that is a function of the PPF’s maximum height ( $h_{max}$ ) and its range which we define by its endpoints a and b:

$$L = \begin{cases} 0 & \text{if } s^i < a \\ h_{max} & \text{if } a < s^i < b \\ 0 & \text{if } s^i > b \end{cases}$$

# Simplest PPF: Rectangle, encompassing the full student distribution

*Simplest PPF where all children learn the same*



$$L = \begin{cases} 0 & \text{if } s^i < a \\ h_{max} & \text{if } a < s^i < b \\ 0 & \text{if } s^i > b \end{cases}$$

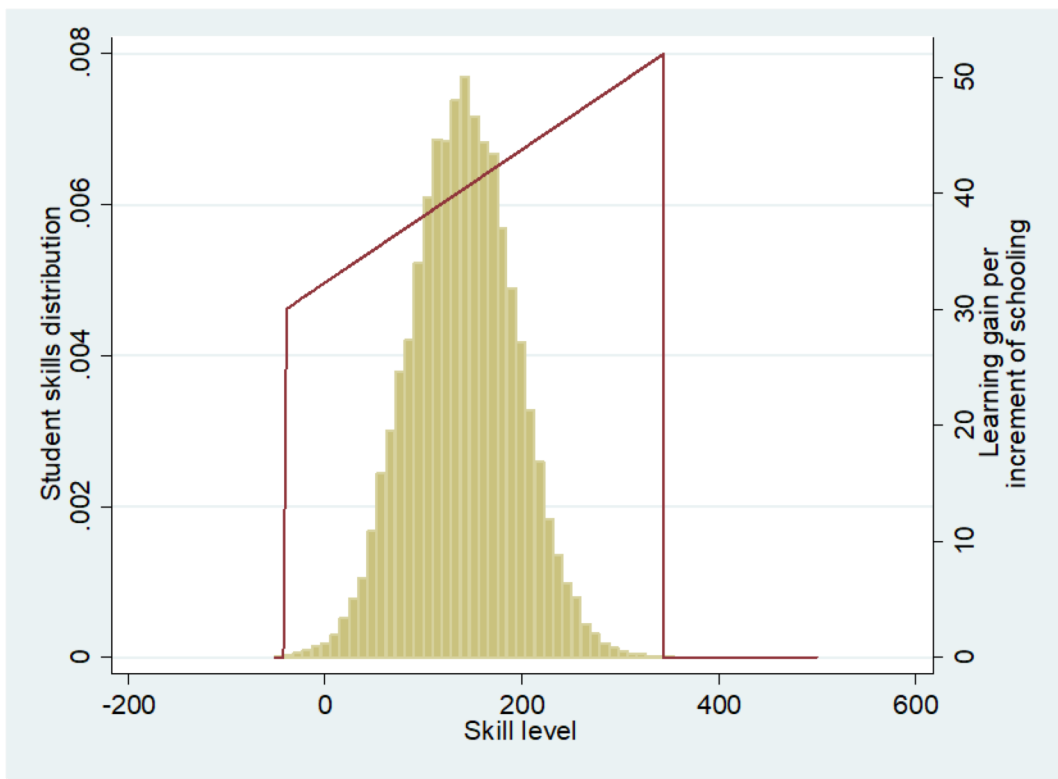
Parameters constructed to encompass full student distribution:

$$\begin{aligned} a &= student_{min} \\ b &= student_{max} \\ h_{max} &= 50 \end{aligned}$$

Illustrated over a student distribution of mean 100 stdev 50

# All children learn a minimum amount, but high performers learn more: Can replicate typical OECD PISA

Instruction increases linearly with initial ability; trapezoidal shape



$$L = \begin{cases} 0 & \text{if } s^i < a \\ h_{min} + (h_{max} * (s^i - a)) / (b - a) & \text{if } a < s^i < b \\ 0 & \text{if } s^i > b \end{cases}$$

Parameters constructed to encompass full student distribution:

- $a = student_{min}$
- $b = student_{max}$
- $h_{max} = 52$
- $h_{min} = 30$

Illustrated over a student distribution of mean 100 stdev 50

# Simulating learning outcomes

- Schooling is a series of instructional processes.
- We use our model to simulate 12 years of schooling:
  - Apply the PPF to the initial student distribution to produce a new student distribution;
  - Shift the PPF at the *pace* of the curriculum to represent instruction at the next grade level;
  - Apply it again;
  - Iterate this 12 time to represent 12 years of schooling
- Produce an average learning profile and disaggregated learning profiles by initial student ability level.

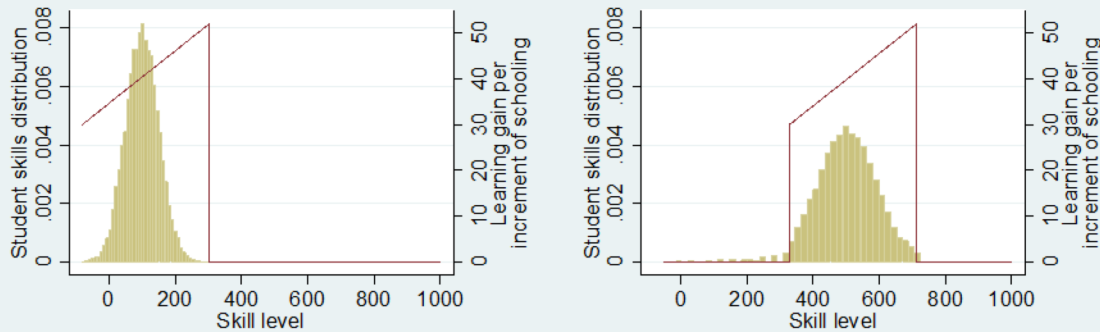
# Simulating learning: how it works

*Students enter Grade 1 with mean score of 100; learn an amount dictated by PPF (maximum of 52, minimum of 30, with an average gain of 42), and then enter Grade 2 with a mean score of 142*

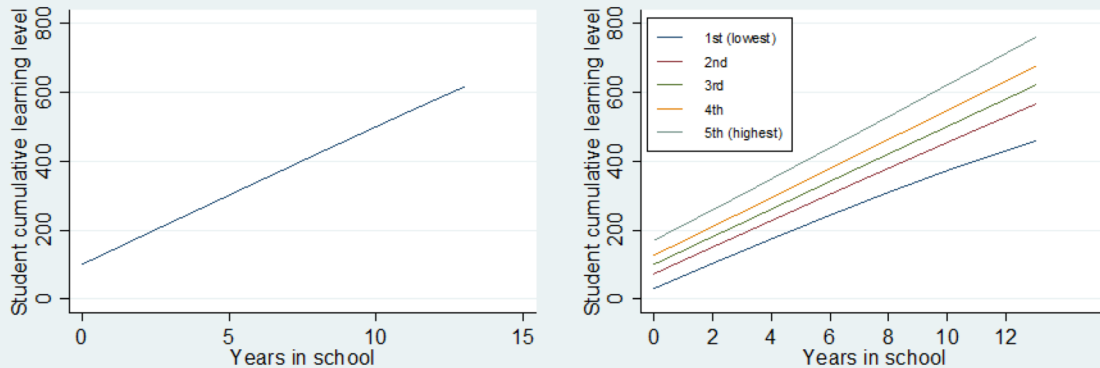


# All children learn a minimum, but high performers learn more, replicating typical OECD PISA scores

Pedagogical function and student distribution at year zero and year 10



Learning profiles: Average, and by quintiles



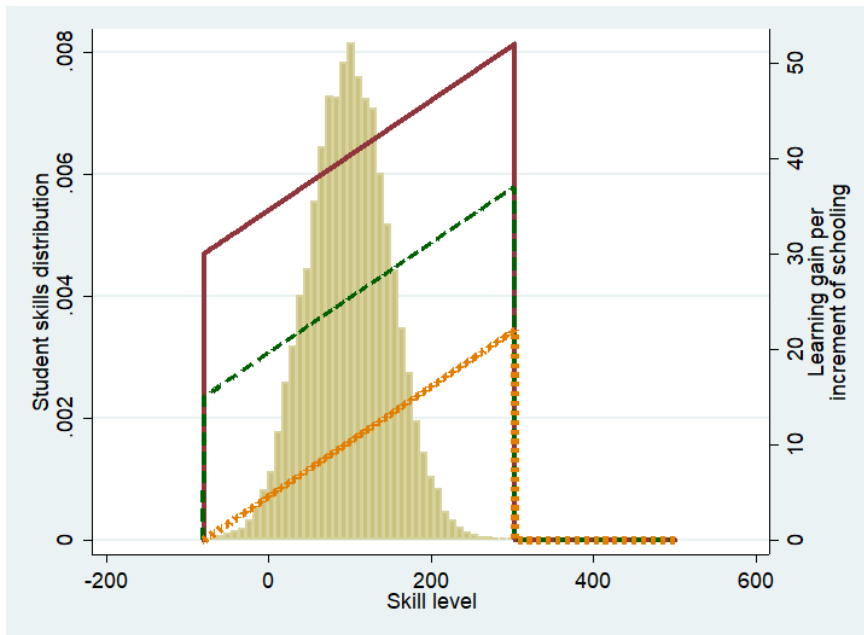
- Parameters calibrated to replicate OECD mean of 500 stdev 100 in grade 10
  - $H_{\max} = 52$
  - $H_{\min} = 30$
  - Pace = 42 (median student keeps pace with curriculum)
- By grade 10 some students are outside range and not learning
- Top quintile learning profile slightly steeper slope – learning more in each grade; bottom quintile slightly flatter slope – learning less in each grade

- PPF as a measure of TVA
- Every teacher has different level of ability – combination of innate ability, training, etc.
- PPF represents teacher’s ability to produce learning outcomes for students at each point in the distribution
  - Max height represents the most learning a teacher is able to produce
  - Range is the range of student abilities for which a teacher is able to produce learning
  - Area under PPF can be thought of as “total ability” to produce learning

# Applications to teacher value-added: Same shape and center, but varied height (learning production) at each point

- PPF as a measure of TVA
- “Typical” understanding of TVA: varying heights, or varying “teacher ability”, varying the total area under the PPF

*3 PPFs of varying height; other parameters constant*



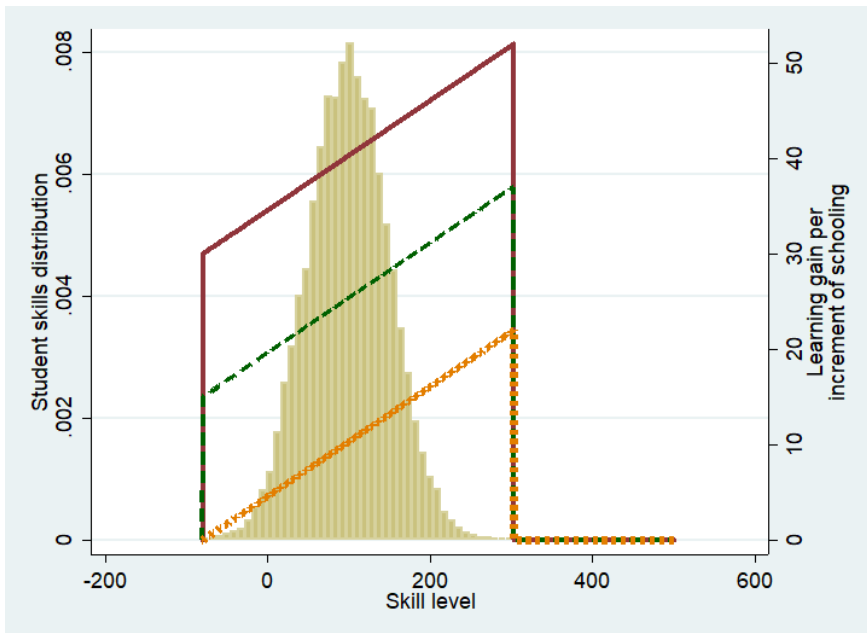
- Red calibrated to replicate OECD scores of mean 500 stdev 100 in Grade 10
- Green and yellow PPFs produce lower learning for students at each point in distribution
  - Green = middle-performing teacher;
  - Yellow = low-performing teacher



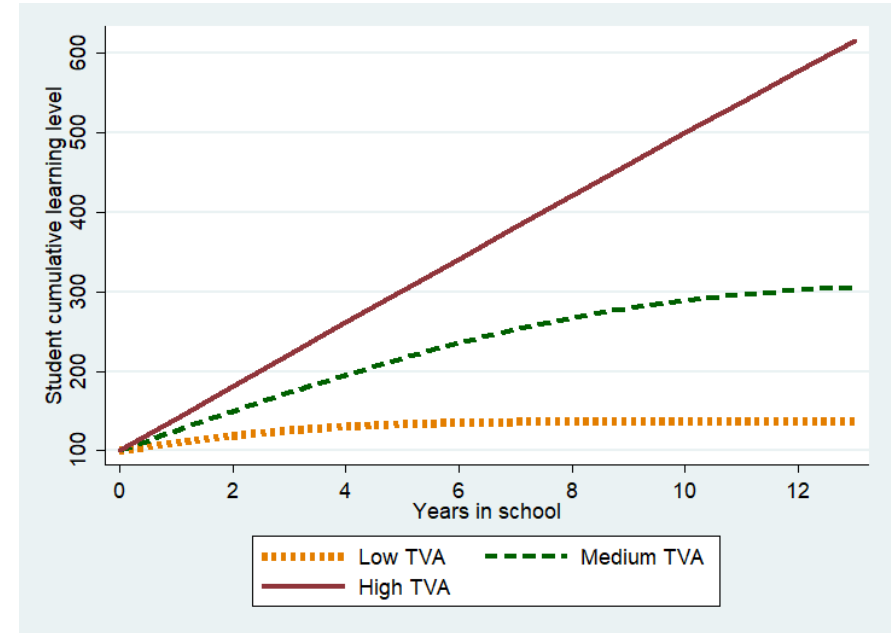
# Applications to teacher value-added: Same shape and center, but varied height (learning production) at each point

- Simulate PPFs across 12 years of schooling to produce learning profiles
- Red replicates OECD; green much lower, yellow extremely low
- If measuring TVA, observe learning levels for a given grade (not PPFs) and assume observing high- middle- and low-performing teachers

*3 PPFs of varying height; other parameters constant*



*Observed TVA: Highest produces high learning, lowest produces very little*

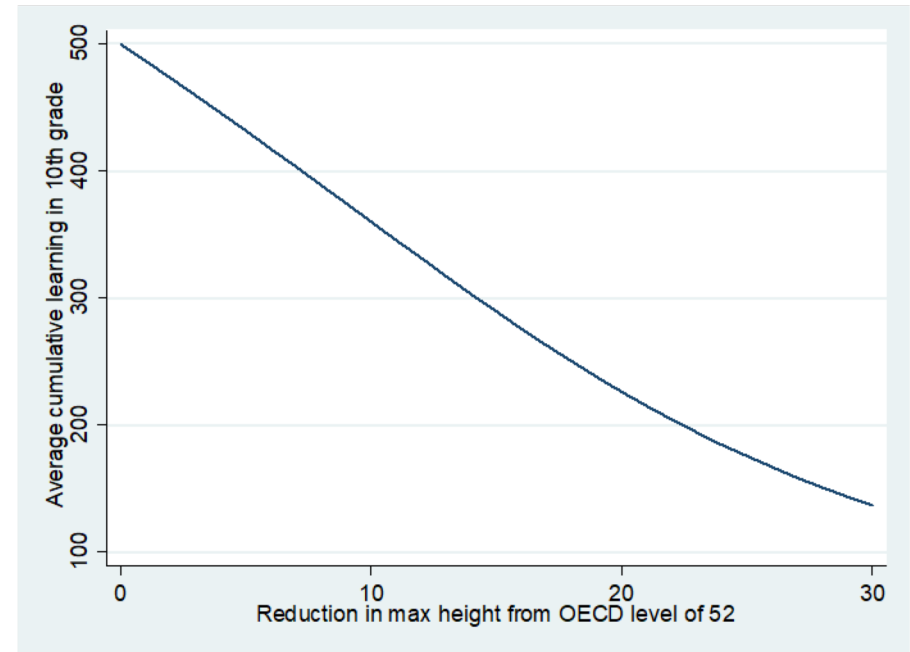
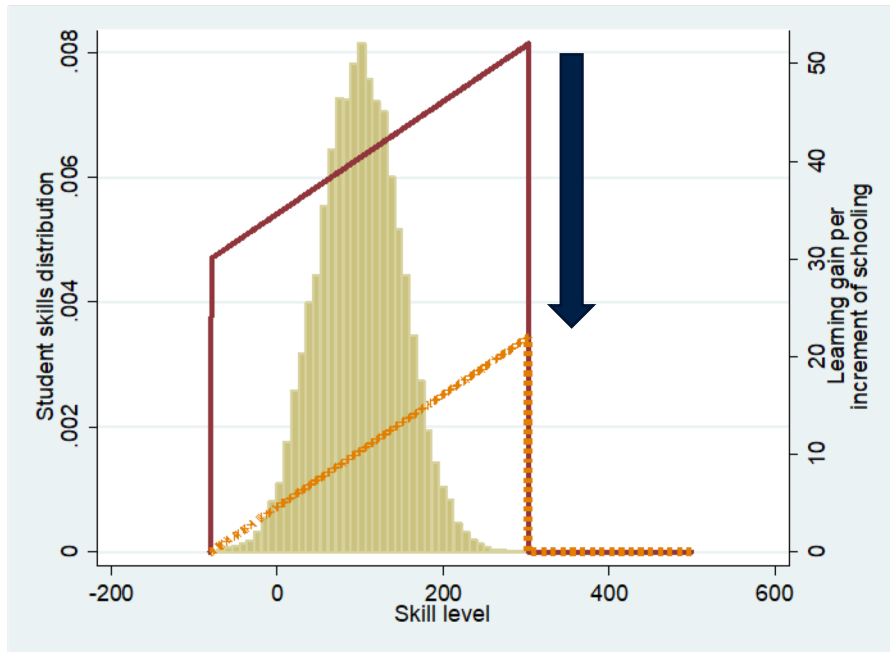


# What if we iterate through all heights within a given range? What average learning levels are produced?

By varying only height (other parameters constant), can produce learning outcomes in Grade 10 of everything from OECD scores to almost no learning at all

*Vary PPF height from max height 52 to max height 22; other parameters constant*

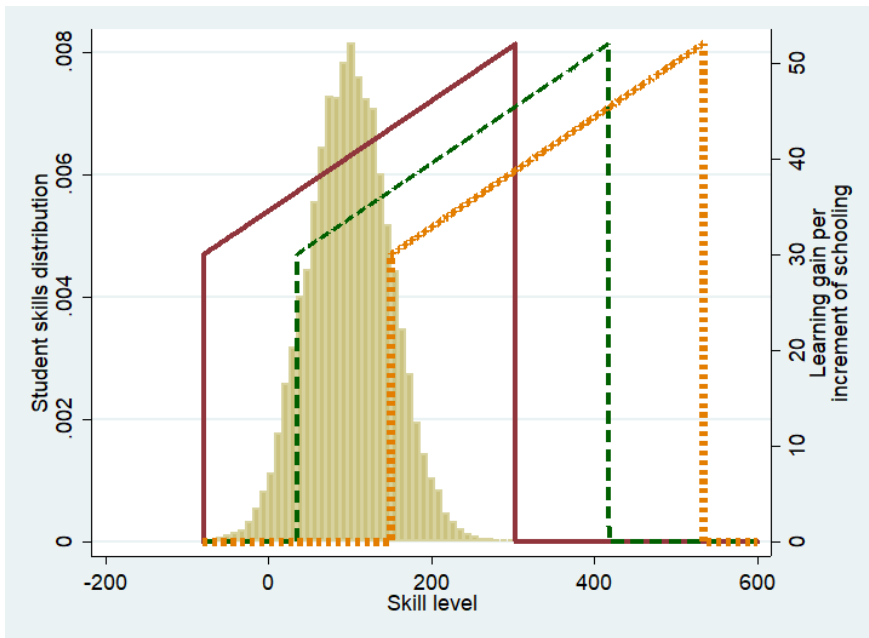
*Starting from OECD average of 500, down to average cumulative learning of less than 150*



# Now shape and size constant, but teaching not aligned with student ability

- 3 PPFs represent teachers of *equal abilities* – same height, same area
- Centeredness of instruction on the student distribution varies (with other parameters constant)
- Could represent overambitious curriculum, methods, etc.

3 PPFs with same area = same “ability”, but with varied centeredness

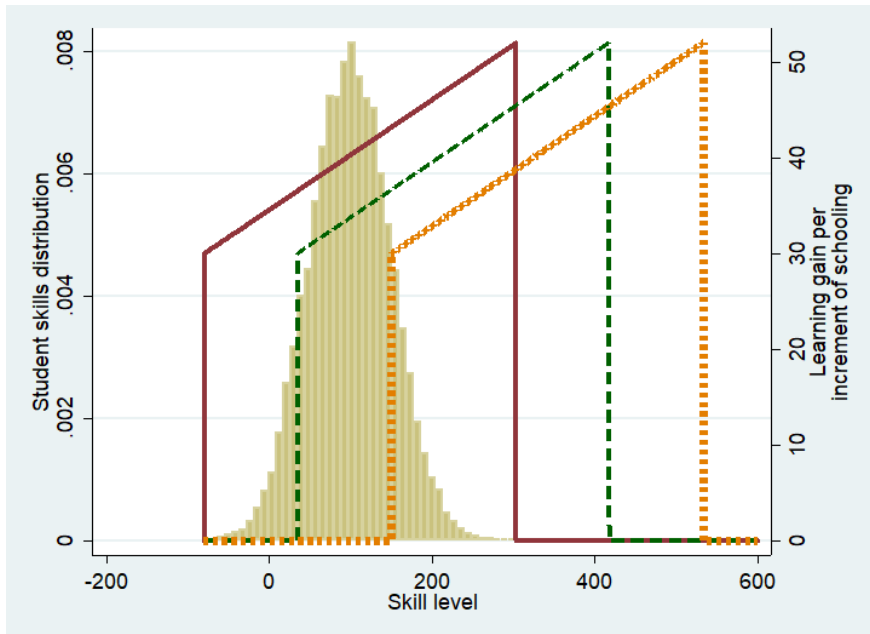


- Red calibrated to replicate OECD scores of mean 500 stdev 100 in grade 10
- Green = same *ability* (height, area) but off-center slightly; using curriculum or methods slightly ahead of student abilities
- Yellow = same *ability* (height, area) but off-center substantially; using curriculum or methods substantially ahead of student ability

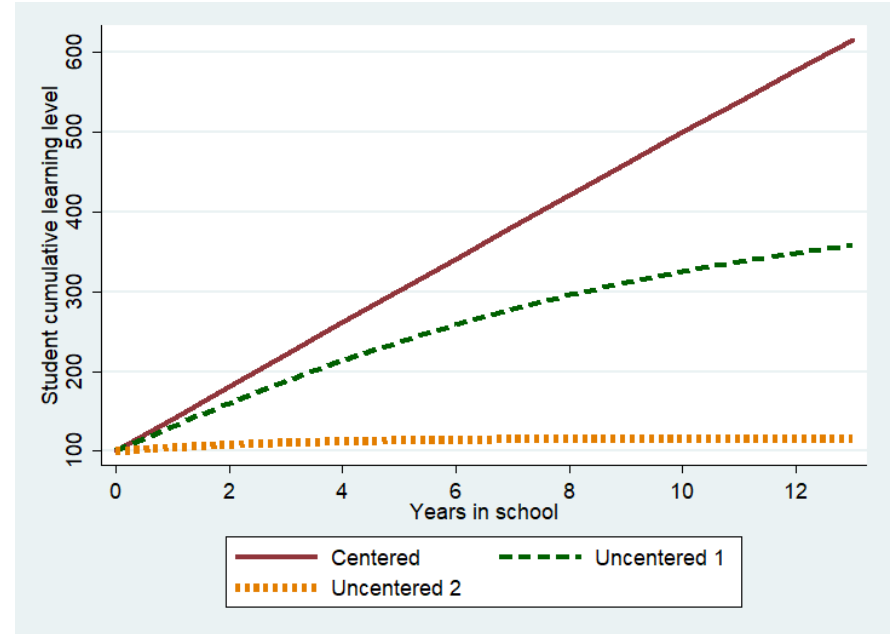
# Now shape and size constant, but teaching not aligned with student ability

- Simulate PPFs across 12 years of schooling to produce learning profiles
- Red replicates OECD; green and yellow have same “ability” as OECD teacher, but instruction isn’t centered, producing substantially less learning
- If measuring TVA, observe learning profiles for a given grade and assume observing high-, middle- and low-ability teachers, when actually ability is constant and centeredness is the problem

*3 PPFs with same area = same “ability”, but with varied centeredness*



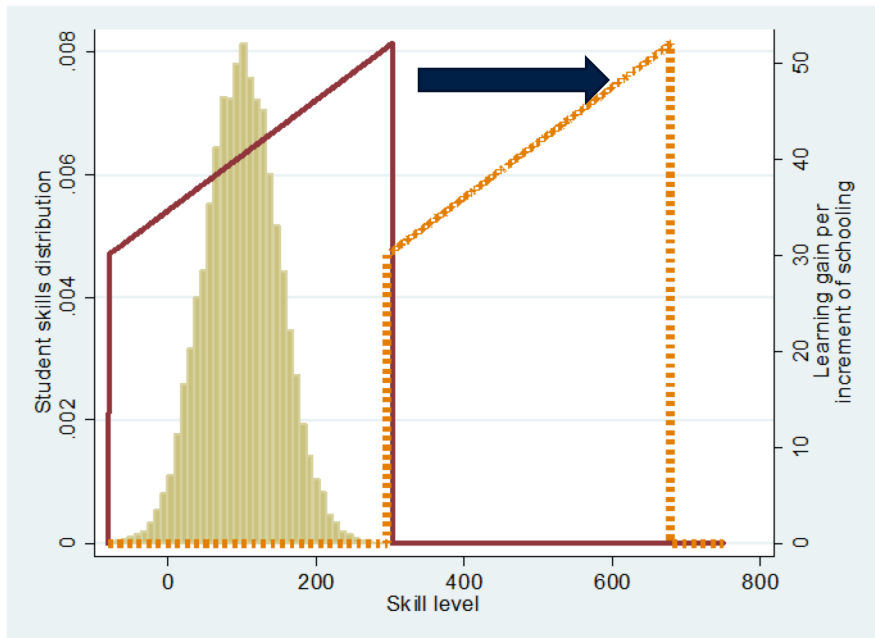
*Observed TVA: Massive variation in learning outcomes for teachers of same potential ability*



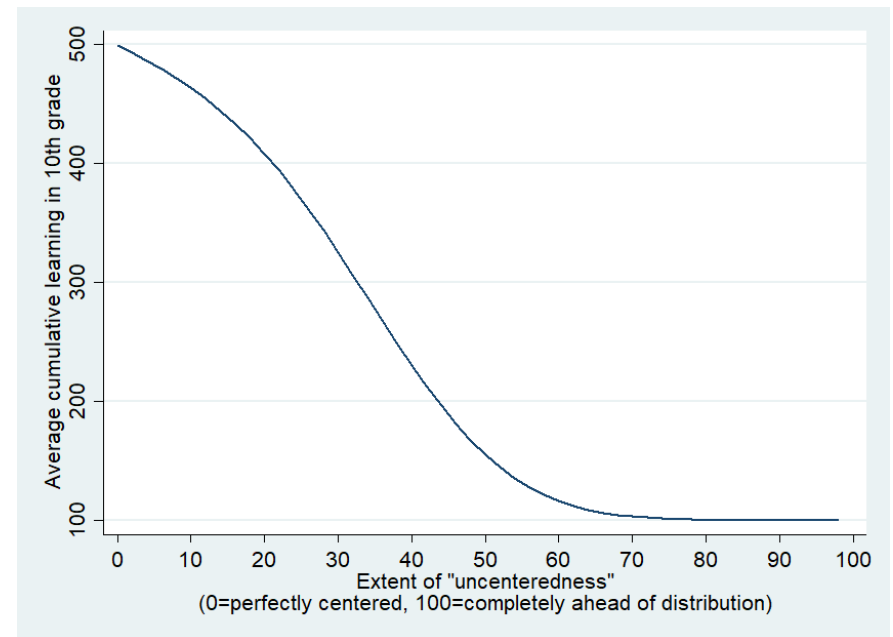
# What if we iterate across all locations within a given range? What average learning levels are produced?

- By varying only centeredness (other parameters constant), can produce learning outcomes in Grade 10 of everything from OECD scores to almost no learning at all

*Vary PPF centeredness from centered to fully uncentered; other parameters constant*



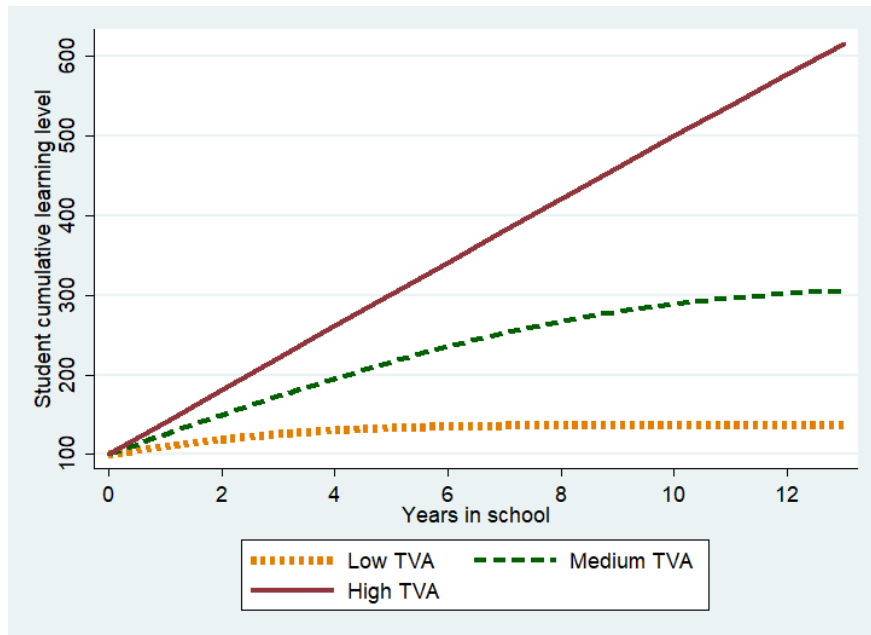
*Average cumulative learning in Grade 10 starts from OECD average of 500, down to average of 100 just by varying centeredness*



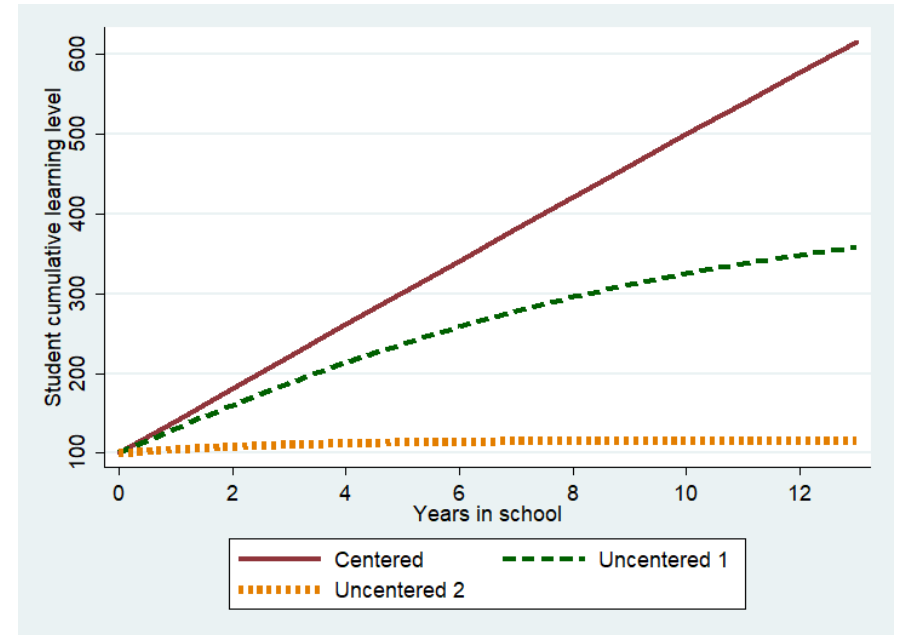
# Observed TVA underdetermines the PPF

Comparing only learning outcomes does not reveal whether teacher effectiveness or another parameter varies.

*Learning profiles varying only height*



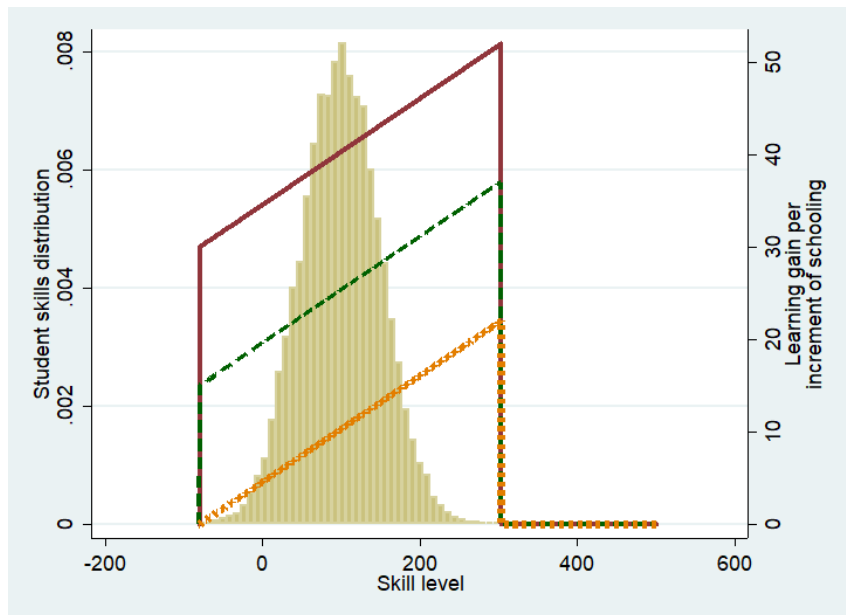
*Learning profiles varying only centeredness*



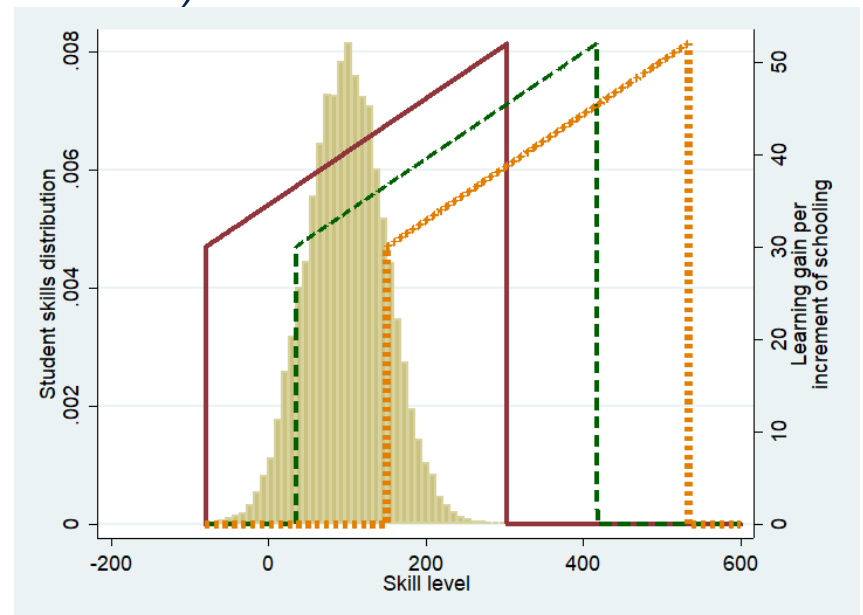
# Improving learning requires improving the right parameter

- A teacher training program aimed at improving teacher ability (height) among low-performing (yellow) teachers could improve learning in Scenario A but not (much) in Scenario B
- Critical to consider appropriate parameter when attempting to improve learning

*A. Increase learning through either improved height (e.g. teacher training) or slower pace (aligning instruction with ability)*



*B. Increase learning through improved centeredness, aligning instruction to student ability (e.g. appropriate curriculum or teaching methods)*



# Conclusions

- Learning outcomes are driven by the initial distribution of student skills and an instructional process that produces some level of learning for a child at each point in distribution
- Our PPF model, with parameters for height, shape, range, centeredness, and pace, can reproduce observed learning outcomes
- Application to understanding of TVA shows it is critical to not just consider one parameter of the PPF – such as “height”
  - Must consider all parameters – centeredness, range, shape, pace
  - Adjust the one(s) most critical for improving learning outcomes



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## Thank you

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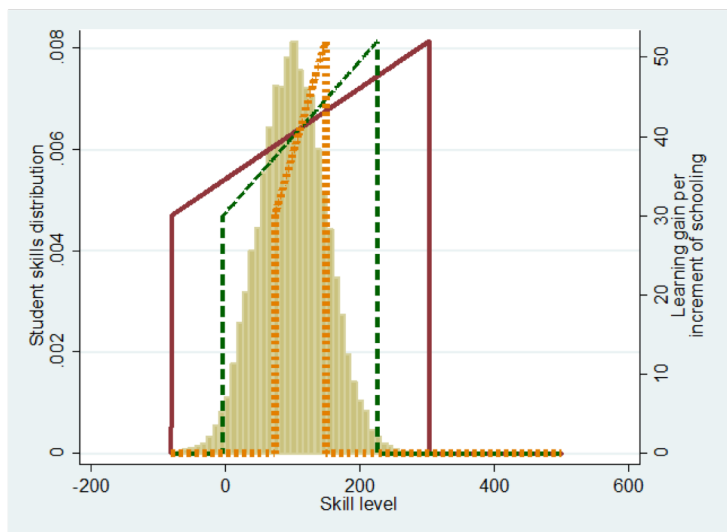


# Appendix: Varying Range

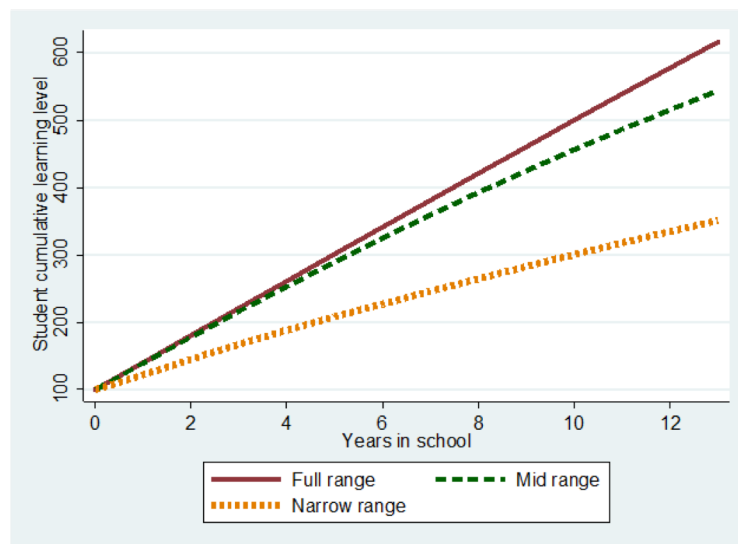
# Same max and min height, but varied range of meaningful instruction

- Some teachers are able to adapt teaching to wide range of student skills, others can only accommodate narrow range
- Simulate PPFs across 12 years of schooling to produce learning profiles
- Red replicates OECD; green and yellow have smaller range of skills they are able to instruct
- If measuring TVA, observe learning outcomes, but not underlying PPF

*3 PPFs with same max and min height, but varied range – e.g. varied ability to teach students at different skill levels*



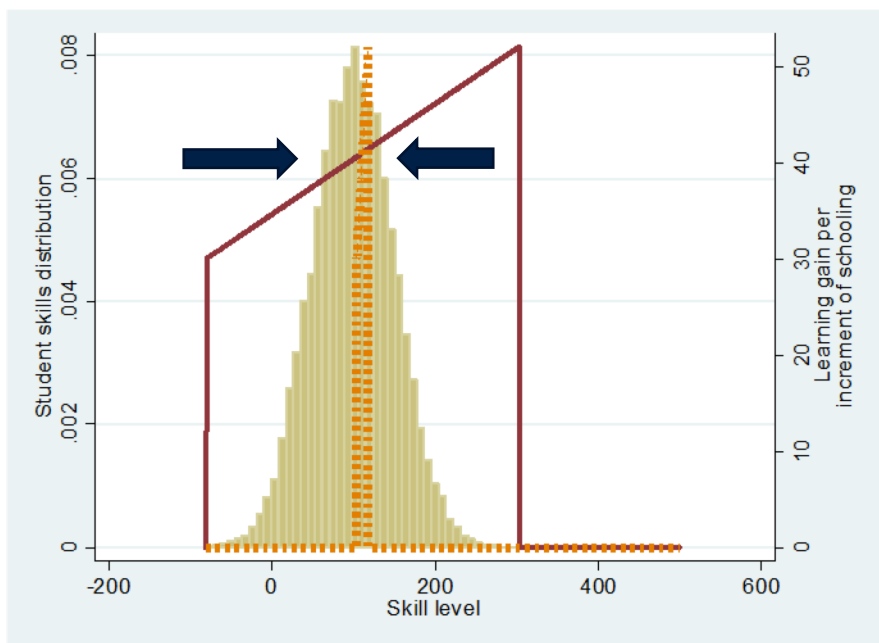
*Observed TVA: Produces increasingly less learning as range shrinks*



# What if we iterate across all ranges, what average learning levels are produced?

- By varying only range (other parameters constant), can produce learning outcomes in Grade 10 of everything from OECD scores to almost no learning at all

*Vary PPF range from full distribution to only center of distribution; other parameters constant*



*Gradual decline then sharp decline as more students fall outside range of PPF*

